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Connected Spaces

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Abstract

Connected spaces are one of the most important mathematical concepts in algebraic topology and differential geometry which are increasingly being used in in distinguishing, describing and classifying various topological spaces. A space is also connected is it exists in one piece and disconnected if it can be written as a union of two separated non empty pieces. Connected spaces a wide range of applications in various fields such as intermediate value theorem particularly when it comes to the Internet of Things (IoT) where it can be used in the design of sensor systems, online screen applications and data trackers among other IoT devices.

1. Introduction

Connected spaces are an important principle in algebraic topology which refers to the topological space which cannot be expressed as a union of 2 or more open and disjoint subsets. The mathematics of connectedness is critically important as it is widely used in distinguishing, describing and classifying various topological spaces. It has a wide range of applications in various fields such as intermediate value theorem [1].

2. Connected Spaces in Topology

The idea of connectedness is particularly premised on the fact that some spaces can effectively be partitioned into various disjoint open sets while other spaces simply cannot be partitioned. It is the spaces which cannot be partitioned which are deemed to be connected spaces [2]. A space is normally considered to be interconnected when a curve can easily be drawn between any given two points within the space. Additionally, a space is also connected is it exists in one piece and disconnected if it can be written as a union of two separated non empty pieces.

The maximal connected subsets for nonempty topological spaces are referred to as the connected components of the particular space [3]. Generally, the components of a topological space X usually form a partition of X when they are nonempty, disjoint and have a union of the entire space. It is considered that every component is a closed subset of the original space. However, this is not the case with infinite numbers [4]. Generally, one of the most important examples of a connected space is a real line in a standard topology with every interval in the subspace topology. On the other hand, a space is considered to be disconnected when all the components of the space are one point sets. The primary theorem of connectedness is that if X and Y are the topological spaces and f=: X → Y is a continuous function, then X will be path connected while image f(X) will be path connected. This is the basis of intermediate value theorem [4].

3. Applications of Connected Spaces

The mathematical principles of connected spaces have a wide range of important applications particularly when it comes to the Internet of Things (IoT). For example, the concept of connected spaces can effectively be used in the design of sensor systems, online screen applications and data trackers among other IoT devices [2]. This will not only allow for the tapping of unlimited data from the internet infrastructure but will also make electronic devices to be more useful and smarter. Theoretically, a greater number of connected spaces will result in more information being readily available thereby allowing various internet based devices to effectively interact in a more accurate way.

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